Solvent Effects on Strong Charge-Transfer Complexes. IV. Trimethylamine and Sulfur Dioxide in the Vapor Phase

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Abstract: Thermodynamic constants for the formation of the 1:1 molecular complex between trimethylamine (TMA) and SO_2 have been obtained from PVT and vapor-density data for equimolar mixtures of these gases in the temperature range 35–44°. Results are in excellent agreement with those obtained previously from measurements of ultraviolet spectra. The present research supports the previous conclusion that the extinction coefficient and oscillator strength of the charge-transfer (CT) band of TMA–SO_2 are nearly the same in the gas phase as in the solvent heptane; in this respect the complex differs from all other CT complexes for which both gas-phase and solution data are available.

In previous publications we have reported thermodynamic and spectral data for the charge-transfer (CT) complex between trimethylamine (TMA) and sulfur dioxide (SO_2) in the vapor phase,[1] in the nonpolar solvent heptane,[1] and in the moderately polar solvents dichloromethane and chloroform.[2] An interesting result of these studies was that the intensity of the CT absorption band changes very little when the medium is changed from vapor to a solvent. This observation is at variance with results obtained from vapor-phase studies of CT complexes with iodine,[3-5] tetracyanoethylene,[5] and carbonyl cyanide[6] as acceptors. In these cases, much lower intensities for the CT band have invariably been found in the vapor phase as compared to solution, both for weak[3] and, with iodine, also for stronger adducts.[4,5] There seems reason, however, to question the reliability of previously reported extinction coefficients of vapor-phase CT complexes.

Unfortunately, only two vapor-phase complexes have been studied independently by different workers.[3-5,7] For the weak benzene–I_2 complex, one report[7] concluded that only the product of the formation constant (K_c) and the extinction coefficient for the CT band (ϵ) could be calculated. In the other case, that of the moderately strong diethyl sulfide–I_2 complex,[4,5] the reported values of ϵ differed by a factor of more than 3, although the ϵK_c product agreed to within experimental error. This is somewhat surprising, since in this case a sufficient amount of the acceptor was apparently in the complexed form at the highest donor concentrations to permit calculation of K and ϵ separately.[8,9] However, random and systematic errors in absorbances are considerably larger for vapor-phase studies than in solution. Moreover, limitations are imposed by the low volatility and instability of reactants and complexes. We feel, therefore, that published extinction coefficients and formation constants for vapor-phase CT complexes should be regarded as highly uncertain. Since it is possible to determine the product ϵK_c for a given complex with reasonable accuracy from vapor-phase spectral data, this situation seems to call for the determination of K_c by nonspectral methods (*e.g.*, from PVT or vapor-density data) whenever it is feasible. With these methods, only one parameter (K_c) has to be inferred from a set of measurements at varying concentrations of the reactants.[10] Consequently, an accurate value of K_c can be deduced for systems in which only a small percentage of either component is complexed, and spectral results for the ϵK_c product then will yield a reliable value for ϵ. To avoid lengthy extrapolations, the classical methods should preferably be employed in the same temperature range as for the spectral study. We have therefore initiated programs in our laboratories aimed at the determination of formation constants for CT complexes in the vapor phase by classical methods. Since the TMA–SO_2 complex is apparently an exception with regard to the relationship between vapor-phase and solution CT absorption intensities, we have studied this complex by two independent nonspectral methods, in the temperature range 35–44°. PVT data found in the literature[11] have been used to calculate K_c at higher temperatures.

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Experimental Section

Apparatus and Experimental Procedure. In one set of experiments, the isothermal expansion of equimolar mixtures of TMA and SO_2 was studied using a Burnett-type apparatus similar to one previously described.[12] The two chambers had volumes of approximately 0.55 and 0.24 l., respectively. The exact volume ratio (the ratio of the large chamber volume to the total volume) was determined with dry nitrogen. This ratio, which is an important parameter in the calculations, was reproduced to better than 1 part in 3000, over the same pressure range as that used in the actual experiments. The pressures were measured using a Texas Instruments fused-quartz precision gage.[12] The apparatus was immersed in a constant-temperature water bath controlled to ±0.01°.

The equimolecular mixture of TMA and SO_2 vapors was transferred to the apparatus from a separate bulb containing the solid TMA–SO_2 addition compound. This compound was prepared essentially as described by Burg.[11] Gaseous SO_2 from a commercial cylinder was admitted into the Burnett apparatus and the pressure recorded. The gas was then frozen with liquid nitrogen into the compound storage bulb. An approximately equal molar amount of gaseous TMA, also from a tank, was then introduced into the apparatus and frozen into the storage bulb, where the addition compound was formed. Pumping for about 10 min with this bulb immersed in a Dry Ice–acetone bath left a white, readily sublimable compound. The dissociation pressure, measured between 25 and 44° agreed to better than 0.2 Torr with the equation $\log P = 11.276 - 3165/T$.[11] Therefore, further purification of the compound was not considered necessary. As noted by Burg,[11] excess of either of the two components could always be pumped out, giving a compound whose dissociation pressure agreed with the values measured after freezing equimolar mixtures of TMA and SO_2. To avoid the necessity of correcting for sorption, the initial pressure in the Burnett apparatus did not exceed 70–80% of the dissociation pressure. In some of the experiments the pressure varied slightly between the final measurement for one expansion and the initial measurement for the next. Measurements were made at 35, 40, and 44°, with initial pressures at approximately 8.0, 13.0, and 16.0 Torr, respectively; 8 to 10 experiments were performed at each temperature, giving 25–30 sets of values for (a) initial pressure in the large bulb, (b) initial pressure in the small bulb, and (c) final total pressure.

In a separate experiment a fused-silica buoyancy microbalance[13,14] was used to determine vapor densities for equimolar mixtures of TMA and SO_2. These mixtures were introduced into the balance chamber (volume *ca.* 3.5 l.) from a storage bulb containing the solid compound (see above). The level of the balance pointer was measured to 0.02 mm with a precision cathetometer, and the corresponding pressures were measured with the Texas Instruments gauge to ±0.003 Torr. A series of measurements was performed with TMA–SO_2 mixtures at 35°, in the pressure range 0–7 Torr. Then pure nitrogen gas was introduced into the system and a series of measurements was made over the same range of vapor densities. A numerical interpolation procedure[14] was used to calculate the pressure of nitrogen corresponding to the same density as that of each TMA–SO_2 sample. This pressure was then converted into a gas density value assuming nitrogen to behave ideally in the pressure range employed.

Methods of Calculation. The PVT data were analyzed using a method similar to that described previously.[12] Equations analogous to (7) and (8) of ref 12 may be derived for a two-component system in which only the 1:1 hetero complex, and no self-associated aggregates, are assumed to be present[15]

$$p = p_{SO_2} + p_{TMA} + K_p p_{SO_2} p_{TMA} \qquad (1)$$

$$\pi_{SO_2} = p_{SO_2} + K_p p_{SO_2} p_{TMA} \qquad (2)$$

$$\pi_{TMA} = p_{TMA} + K_p p_{SO_2} p_{TMA} \qquad (3)$$

where p_{SO_2} and p_{TMA} are the monomer pressures of SO_2 and TMA, respectively, and K_p is the formation constant in reciprocal pres-

sure units. Equations 2 and 3, the formal pressure equations, are identical term by term, since equimolar mixtures were employed throughout. Therefore, it was possible to analyze the PVT data by treating TMA–SO_2 as a pseudo-one-component system. The nonlinear least-squares technique of Tucker, *et al.*,[12] was used directly to infer values of the formation constant from sets of pressure values measured with the Burnett-type apparatus.

Equations 1–3 are also applicable to the vapor-density data, and in fact π_{TMA} and π_{SO_2} may be calculated directly from the measured vapor densities. To avoid systematic errors which would be introduced if the mole ratio of the TMA–SO_2 mixture deviated even slightly from 1:1, a difference method was employed to infer values of K_p from sets of values of p and π (where $\pi = \pi_{SO_2} + \pi_{TMA}$). Using a trial value of K_p, the monomer pressures corresponding to a given pair of initial and final pressures were computed. The anticipated value of the change in formal pressure, $\Delta\pi_{calcd}$, could then be computed (using the monomer pressures and the same value of K_p) and compared to the observed $\Delta\pi$. The procedure was repeated for each pair of initial and final pressures and the error function

$$RMSD = \left\{ \sum_{i=1}^{n} (\Delta\pi - \Delta\pi_{calcd})^2 / (n - 1) \right\}^{1/2} \qquad (4)$$

was calculated corresponding to the chosen value of K_p (n is the number of $\Delta\pi$ values). A nonlinear least-squares method was used to minimize the RMSD with respect to the single unknown parameter K_p.[12,16]

Results and Discussion

Formation constants for the TMA–SO_2 1:1 complex obtained from the Burnett-type experiment at 35, 40, and 44° are listed in Table I. Standard errors in the

Table I. Formation Constants for the Trimethylamine–Sulfur Dioxide Complex in the Gas Phase

Temp, °C	K_p, Torr^{-1}	RMSD, Torr	K_c, l./mol	Method
35.0	0.00946 ± 0.00057	0.0051[a]	182 ± 11	Burnett
40.0	0.00738 ± 0.00033	0.0063[a]	144 ± 6	Burnett
44.0	0.00550 ± 0.00031	0.0098[a]	109 ± 6	Burnett
35.0	0.0106 ± 0.0019	0.035[b]	203 ± 36	Vapor density
39.7			154 ± 12	Spectral[1]

[a] Root-mean-square deviation in final pressure. [b] Root-mean-square deviation in $\Delta\pi$ (see eq 4).

formation constants and values of the root-mean-square deviation in pressure are given. Table I also includes K_c calculated from the vapor-density data at 35° and K_c at 39.7° obtained from the spectrophotometric study reported before.[1] Table II gives the experimental data at one temperature (40°) for the Burnett experiment. It is seen that the agreement between calculated final pressures and observed pressures is extremely good, giving small standard errors in K_c. For the vapor-density measurements, the standard error was considerably larger.

The vapor pressure predicted for complete dissociation of a given weight of the solid TMA–SO_2 adduct, assuming ideal gas behavior, can be inferred from the PVT data in ref 11, for the temperature range 65–100°. The formation constants at the various temperatures can then be calculated, attributing the difference between the calculated and observed pressures to complex formation. The results are given in Table III.

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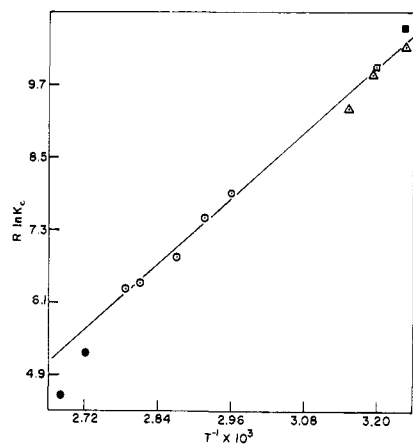
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Figure 1. Van't Hoff plot of temperature dependence of the formation constant (K_c) for TMA–SO$_2$ from four different techniques: O, PVT (ref 11); △, PVT, Burnett type (this work); □, spectral (ref 1); ■, vapor density (this work); ●, PVT (ref 11), not included in least-squares fit.

Since one Burnett-type experiment was performed at nearly the same temperature (40°) as that used in the spectrophotometric study (39.7°), the formation constants obtained by these methods can be directly compared. The value of 144 ± 6 l./mol from the Burnett

Table II. Measured (PL1, PS1, and P2) and Calculated (P2C) Pressures of TMA–SO$_2$ Mixtures at 40° from Burnett Experiments[a]

PL1, Torr	PS1, Torr	P2, Torr	P2C, Torr[b]
9.750	0.077	6.797	6.794
6.800	0.057	4.728	4.733
4.728	0.024	3.821	3.282
11.518	0.067	8.025	8.025
13.196	0.120	9.228	9.215
9.231	0.084	6.434	6.434
6.436	0.057	4.470	4.479
4.471	0.043	3.109	3.110
13.042	0.117	9.111	9.105
9.111	0.073	6.347	6.347
4.410	0.046	3.064	3.068
11.203	0.095	7.806	7.813
7.808	0.048	5.427	5.431
5.428	0.033	3.765	3.772
11.040	0.057	7.691	7.688
7.691	0.060	5.348	5.354
5.350	0.041	3.809	3.719
11.654	0.063	8.128	8.119
8.128	0.052	5.652	5.655
5.654	0.038	3.921	3.930
10.896	0.090	7.592	7.597
7.592	0.054	5.275	5.283
5.275	0.035	3.661	3.666

[a] PL1, PS1, and P2 represent, respectively, the initial gas pressure in the large bulb, the initial pressure in the small bulb, and the final pressure in both bulbs at equilibrium. See ref 12 for details of the experiment and treatment of data. [b] Apparatus volume ratio (ratio of volume of large bulb to the total system volume) equals 0.6911.

method is in good agreement with the result 154 ± 12 l./mol from the spectral measurements. To demonstrate the consistency between the formation constants determined by all of the four methods at various temperatures, a plot of $R \ln K_c$ vs. $1/T$ is shown in Figure 1. The amount of complex present in the PVT experiments was very small at the two highest temperatures, and the

Table III. Deviations from Ideality and Calculated Formation Constants from PVT Data for TMA–SO$_2$ (ref 11)

Temp, °C	Pressure, Torr			$K_p \times 10^3$, Torr^{-1}	K_c, l./mol
	Exptl	Calcd	Complex		
64.7	77.6	81.1	3.5	2.56	53.9
69.9	79.1	82.1	3.0	2.06	44.0
75.1	80.85	83.9	2.24	1.45	31.5
82.8	179.7	189.9	10.2	1.14	25.3
85.7	83.8	85.6	1.8	1.07	24.0
94.5	86.5	87.6	1.1	0.60	13.8
99.9	88.2	89.0	0.8	0.42	9.8

calculated formation constants cannot be considered very reliable. When these two K_c values are omitted, all the other data fit nicely to eq 5, which was obtained by least-squares analysis

$$R \ln K_c = \frac{(9.07 \pm 0.27) \times 10^3}{T} - (18.9 \pm 0.80) \quad (5)$$

where R is 1.987 cal mol^{-1} deg^{-1}. This gives for the complex formation reaction in the gas phase $\Delta U° = -9.1 \pm 0.3$ kcal and $\Delta S° = -20.9 \pm 0.8$ eu, where $\Delta U°$ and $\Delta S°$ are internal energy and entropy changes for formation of 1 mol of TMA–SO$_2$ from 1 mol each of TMA and SO$_2$, using unit molarity, ideal gas standard states for each species. Including all the points gives $\Delta U° = -9.7 \pm 0.4$ kcal, $\Delta S° = -22.9 \pm 1.0$ eu. These values may be compared with the spectrophotometric results $\Delta U° = -9.1 \pm 0.4$ kcal, $\Delta S° = -21.0 \pm 1.0$ eu.[1]

Figure 1 shows the excellent agreement between the value of $R \ln K_c$ at 39.7° from the spectrophotometric study and that calculated from the least-squares line ($R \ln K_c$ vs. $1/T$) obtained from the nonspectral values of K_c. Moreover, the excellent agreement between the value of $\Delta U°$ calculated from the change in intensity of the CT band with temperature and the value reported here must be considered fortuitous. Since the inherent problem connected with the spectral study in the vapor phase is the calculation of separate values of K_c and ϵ, whereas the product $K_c\epsilon$ can be determined accurately, it follows that the previously reported value for ϵ is reliable. The precision and internal consistency of spectral results for the TMA–SO$_2$ system in condensed phases[1,2] lead us to believe that reported solution values of ϵ and the oscillator strength are accurate. The present study therefore confirms the important observation that the spectral parameters of the CT band are nearly the same in the vapor phase as in solution.

Various attempts to rationalize the increase in vapor-phase extinction coefficients as compared to those evaluated in solution have been discussed in several papers concerned with vapor-phase CT complexes.[1,3-6] Some of the proposed mechanisms[6,17-19] may explain the increase for weak complexes, but these effects should be less important as the donor–acceptor interaction becomes stronger. Nevertheless, a drastic change in the value of ϵ is reported for the moderately strong organic sulfide–iodine complexes.[5] It remains a perplexing problem why two complexes of comparable

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strength (in terms of ΔU) such as tetrahydrothiophene–I_2[5] and TMA–SO_2 should behave so differently in this respect.

We believe that further progress toward understanding solvent effects on the spectral parameters of CT complexes will depend upon two factors: (1) the acquisition of additional accurate data for both weak and strong complexes in the vapor phase; and (2) development of methods for the study of particularly weak complexes in solution, which may remove possible inadequacies in the interpretation of the solution data.[20] The present study demonstrates the value of a combination of

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spectral and nonspectral methods to obtain reliable spectral parameters in the vapor phase. For systems in which donor, acceptor and complex are not very volatile, additional techniques must be used. One new method, employing a mixture of polyiodides as a constant iodine activity source,[21] has been successfully used to study the diethyl ether–iodine adduct; the technique is apparently applicable to systems in which either very weak or moderately strong complexes are present.

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Statistical Theory of Cooperative Binding to Proteins. The Hill Equation and the Binding Potential[1]

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Abstract: The Hill equation and the binding potential are useful methods of describing the cooperative binding of ligands to proteins. Starting from the formal theory of solutions developed by McMillan and Mayer, statistical mechanical versions of both of these classical expressions are here derived. The fractional occupation of protein sites by ligand is expanded in powers of protein concentration, and is used to derive explicit expressions for the effects of intermolecular (protein-protein) interactions on both the Hill equation and the binding potential. The theory leads to a new definition of the apparent free energy of interaction between sites on a single molecule in terms of the free energy of a ligand-transfer reaction between two macromolecules, and provides insight into the significance of the slopes of Hill plots in terms of ligand-transfer processes. The results indicate that, in most cases, the Hill plot parameters may be expected to be influenced to only a minor or negligible extent by intermolecular forces.

The object of this paper is to discuss an empirical equation of A. V. Hill[2] and the binding potential of Wyman[3] from the point of view of McMillan–Mayer[4] solution theory. The Hill equation, which has often been used to describe binding of ligands to proteins,[5] can be written as

$$\bar{Y}_i/(1 - \bar{Y}_i) = Ka_i^n \qquad (1)$$

where $\bar{Y}_i$ is the fractional occupation of i sites by ligand i at ligand activity a_i. The exponent n is, in general, a function of a_i, but is usually found to be essentially constant over a fairly wide range around the midpoint of the binding curve.[5]

As originally presented by Hill, this equation was a partially successful attempt to describe the cooperative binding of oxygen to hemoglobin. Its modern importance is derived from the demonstration that the Hill equation yields useful thermodynamic information

about homotropic[6] reactions, of whatever origin, in *any* system.[5]

If logarithms are taken of both sides of this equation, the resulting expression forms the basis for the well-known Hill plot.[5] Two important quantities can be extracted from such a plot: (1) the minimum value for the decrease in work per site required to saturate the macromolecule with ligand, which results from cooperative interactions among the sites; and (2) the slope of the Hill plot, n, at the midpoint of titration, which is a measure of the cooperativity of the binding reaction. The two quantities are closely related, and, in fact, one is a function of the other.[5] The determination of the first of these two quantities from a Hill plot and its connection with the second are not based on the assumption of a particular cooperative model, but arise from general thermodynamic considerations.[5]

In the following sections, the methods of statistical mechanics are used to investigate more closely the significance of the two quantities which are derived from Hill plots. The approach taken is, in essence, an extension of the elegant theory of protein solutions pub-

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